Reg. No. : | | | | | | | | | | | |

Question Paper Code : 53243

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

First Semester

Civil Engineering

MA 6151 – MATHEMATICS – I

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

2. Write down the quadratic form corresponding to the matrix $A = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 0 & 3 \\ 5 & 3 & 4 \end{pmatrix}$.

3. Examine the nature of the series $1 + 2 + 3 + 4 + \ldots + n + \ldots \infty$.

4. State the Leibnitz's rule.

5. Show that the family of straight lines $2y - 4x + a = 0$ has no envelope, where 'a' is a parameter.

6. Define the following terms : Radius of Curvature, Center of curvature.

7. State two important properties of Jacobians.

8. Write the formula for Taylor's expansion of $f(x, y)$ about the point (a, b) upto second degree terms.

9. Evaluate $\int_0^1\int_0^1\int_0^1 xyz\,dx\,dy\,dz$.

10. Change the order of integration in $\int_{-2}^{1}\int_{x^2+4x}^{3x+2} dy\,dx$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find all the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$. (10)

(ii) Using Cayley-Hamilton theorem, find the inverse of the matrix $A = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{pmatrix}$. (6)

Or

(b) Find the orthogonal transformation which transforms the quadratic form $x^2 + 3y^2 + 3z^2 - 2yz$ to canonical form. Also determine the index, signature and nature of the quadratic form. (16)

12. (a) (i) Prove that the Geometric series with common ratio 'r' is convergent if $|r| < 1$, divergent if $r \geq 1$ and oscillatory if $r \leq -1$. (8)

(ii) Discuss the series for convergence : $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots \infty$. (8)

Or

(b) (i) Test the series for convergence : $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \ldots$ by comparison test. (8)

(ii) Test the convergence of the series : $\frac{1}{6} - \frac{2}{11} + \frac{3}{16} - \frac{4}{21} + \frac{5}{26} - \ldots \infty$. (8)

13. (a) (i) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (10)

(ii) Show that the radius of curvature of any point (x, y) of the rectangular hyperbola $xy = c^2$ is given by $\rho = \frac{(x^2 + y^2)^{3/2}}{2c^2}$. (6)

Or

(b) (i) Find the center and circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point $\left(a/4, a/4\right)$. (8)

(ii) Find the evolute of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ considering it as the envelope of it normals. (8)

14. (a) (i) If $u=\frac{x+y}{1-xy}$ and $v=\tan^{-1}x+\tan^{-1}y$, find $\frac{\partial(u, v)}{\partial(x, y)}$. Find also a relation between u and v, if it exists. (8)

(ii) Using Taylor's series, expand $\sin x \sin y$ in powers of x and y upto the terms of third degree. (8)

Or

(b) (i) Find the dimensions of the rectangular box without a top of maximum capacity, whose surface area is 108 sq.cm. (8)

(ii) If $z=x^y+y^x$, then prove that $\frac{\partial^2 z}{\partial x\,\partial y}=\frac{\partial^2 z}{\partial y\,\partial x}$. (8)

15. (a) (i) Change the order of integration and evaluate $\int_0^1\int_0^{\sqrt{1-y^2}}\frac{\cos^{-1}x}{\sqrt{1-x^2}\sqrt{1-x^2-y^2}}\,dx\,dy$. (8)

(ii) Find by triple integral, the volume of the tetrahedron bounded by the coordinate planes and the plane $x+y+z=1$. (8)

Or

(b) (i) Evaluate, through the change of variables, the double integral $\iint_R (x+y)^3 e^{-(x-y)}dx\,dy$ where R is the square with vertices (1, 0), (2, 1), (1, 2) and (0, 1) using the transformation $u=x+y$ and $v=x-y$. (8)

(ii) Find the area between the parabolas $y^2=4ax$ and $x^2=4ay$. (8)